# The Heston Model and the Brazilian Options Market

**Abstract**

This work aims to conduct a comparative analysis between the pricing models of Black and Scholes (1973) and Heston (1993) to identify which best embodies the reality of the Brazilian options market. As such, a set of options with underlying shares that comprise the Ibovespa (*São Paulo Stock Exchange Index*) was selected at the time of data collection (November 2014). The share data cover the period from January 2000 to November 2014. For the Heston model, two volatility estimation alternatives were addressed: estimations by *Generalized Autoregressive Conditional Heteroskedasticity* (GARCH) and *General Exponential Autoregressive Conditional Heteroskedasticity* (EGARCH). The results were analyzed by comparing three error indicators: *Mean Absolute Error* (MAE), *Mean Absolute Percentage Error* (MAPE), and *Mean Squared Error* (MSE). After performing this empirical study for each selected option, the Heston model was found to be the best alternative for options pricing.

**Keywords**: shares; options; pricing

## 1. Introduction

The increased use of option and swap contracts during the 1990s brought significant complexity to the study of this financial market segment. Although the success of this type of operation has become indisputable, considerable research should be conducted with respect to the pricing of such assets.

In the development trajectory of the derivatives market, the Black and Scholes (1973) model appears to be of great importance, given that it represents a milestone associated with all proposals for existing price modeling up until its creation. What differentiates this model from others is the simple and practical approach integrated with the agility achieved through the application and ease of interpreting results. This advantage of the Black and Scholes model (B-S) can be attributed to its assumptions that lead to an ideal theoretical setting for its use. However, the introduction of the model in the day-to-day function of the market revealed the weaknesses of the adopted assumptions and gave rise to the main criticisms against the proposed tool. At the heart of these critiques are discussions regarding volatility that, unlike the provision of the constraints of the model, are not constant.

In fact, many research efforts have shown that the calculation of implied volatility, i.e., the volatility found by resolving the B-S model with all other variables observed in the market, results in a "U"-shaped curve. This phenomenon is also known as the "volatility smile" or "the implied volatility smile."

Given the limitation of the B-S model, new studies were conducted to capture the real behavior of the volatility of underlying assets, with the assumption that stochastic processes are adopted. In this new category of models, the works of Hull and White (1987 and 1988), Bates (1996), and Heston (1993) can be highlighted. The Heston model has enjoyed greater popularity than other works that model volatility through stochastic processes. This popularity can be attributed to the capability of the Heston model to provide closed-form solutions for European options. Such form speeds up the calibration process of its parameters and provides a great advantage over other stochastic volatility models. Thus, the model implicitly adapts to market volatility. Heston (1993) also proposes that the return from the underlying assets follows a log-normal distribution and considers the property of reversal relative to the average volatility, contrary to what is considered in the B-S model.

Thus, following the latest line of studies concerning the derivatives market (MORAES et al., 2013 and KAHL et al., 2005), this paper aims to present some of the new and important concepts regarding the modeling intended for pricing options, as well as to test the predictive ability of such concepts in the context of the Brazilian market. In practice, the Heston model will be applied to options traded in Brazil, and the results will be compared with those of the B-S model, which is disseminated as the market model. This is an important and unique characteristic and a contribution of this study, given that no other works have been found to provide this type of application to the Brazilian stock options market.

This research also intends to resume discussions about the importance of the choice of the pricing model and its impact on the price estimation of various kinds of options. In light of these results, the conclusions and suggestions for future research on the subject will be drawn.

# 2. Theoretical Context

## 2.1. Literature Review

The model proposed by Heston (1993), which will be widely covered in this work, has its roots in publications by Black and Scholes (1973), as well as in subsequent contributions by Merton (1973). These publications gave birth to the main instrument that is still adopted today as the basis of modeling for pricing options. Despite their theoretical and academic nature, these seminal works have become fundamental to the day-to-day functions of trading desks and have facilitated the development of many other studies that followed the line of options and other derivative contracts (VOLCHAN, 1999).

In their work, Black and Scholes mentioned some options pricing attempts that had arisen at the time. These attempts include the models by Sprenkle (1961), Samuelson (1965), and Chen (1970). However, they stressed that no attempt had achieved relevant results to justify the applicability of such models in trading desk routines. To overcome this deficiency, much of the market based pricing on the value of the securities involved in each operation or through models that left room for different interpretations (BLACK and SCHOLES, 1973).

Although the success of the B-S model cannot be challenged because of its great contribution, much criticism has been directed toward its restrictions in practical application. In particular, the phenomenon of the volatility smile appears as the main offender to the premise of constant volatility assumed by Black and Scholes in 1973. The implied volatility calculation based on the B-S model reveals that the market assigns different volatilities to identical options with different strike prices and maturities (CARR and MADAN, 1998). The "U" form of the implied volatility curve inspired the name "volatility smile" or "implied volatility smile" and gave rise to a number of subsequent works that sought to develop models that consider this effect when estimating the price of options purchases and the sale of shares (VIANA, 1998).

Authors such as Heston (1993), Bates (1996), and Hull and White (1987 and 1988) sought to solve the problem of volatility estimation by creating models based on the adoption of stochastic processes to estimate the volatility curve. This work will focus on the solution proposed by Heston and the unique characteristics of its results relative to the market model (B-S).

## 2.2. Heston Model and Parameters

The Heston model for stock options can be specified by the following differential equations (HESTON, 1973):

$$\frac{dS(t)}{S(t)} = \mu dt + \sqrt{V(t)}dW_1 \quad (1)$$

$$dV(t) = \kappa\big(\theta - V(t)\big)dt + \sigma\sqrt{V(t)}dW_2 \quad (2)$$

$$dW_1.dW_2 = \rho dt \quad (3)$$

where S(t) is the price in relation to the share at the time t;

μ is the average return on the shares;

V(t) is the variance of stock returns;

θ is the estimated long-term variance;

κ is the speed of reversion of the volatility estimated on average over the long term;

σ is the volatility of volatility;

$W_{1,2}$ are geometric Brownian motions; and

ρ represents the degree of correlation between $W_1$ and $W_2$.

Similar to the model proposed by Black and Scholes in 1973, the Heston model adopts the variables S, V, and μ, which dispense with estimation efforts because they are perceived by the market. The parameters (θ and κ) aim to address the problem of the "volatility smile." In turn, $W_{1,2}$ are geometric Brownian motions that respectively represent the uncertainty of price and the volatility of stock. The parameter ρ indicates the degree of correlation between the Brownian motions, thereby allowing a leveraging effect in the model when $\rho \neq 0$ (MIKHAILOV, 2003).

To make the model more practical, Heston and Nandi (2000) proposed the adoption of a model by Generalized Autoregressive Conditional Heteroskedasticity [GARCH (p, q)] to estimate the long-term series of volatility. According to the authors, the adoption of the GARCH model does not generate significantly different results but allows for easier estimation through a simple observation of price history.

# 3. Methodology

## 3.1. Data and Sample

Seeking to apply the Heston model to the Brazilian scenario of stock options and to compare the results with the prices estimated by the traditional B-S model, the data on options and shares traded on the BOVESPA (São Paulo Stock Exchange) during the period from January 03, 2000 to November 21, 2014 were used. In doing so, call options for which the underlying shares comprised the IBOVESPA at the time of data extraction were selected (November 23, 2014). Employing this series requires the creation of a volatility projection that will be used in the Heston model. Furthermore, the parameters $\kappa$ and $\theta$ will be produced based on these data. The considered prices correspond to the values recorded at the close of each session. This study only considered the options for the seven preferred shares that displayed the largest number of observations in the sample. The selection consisted of the preferred shares from the companies Usiminas S/A (USIM5), Eletrobras S/A (ELET6), Investimentos Itaú S/A (ITSA4), Cemig S/A (CMIG4), Petrobras S/A (PETR4), Vale S/A (VALE5), and Gerdau S/A (GGBR4), all of which correspond to a total of 29,251 observations.

Table 1 – Identified roles and observed trading days

| Role Code | Observations |
|---|---|
| USIM5 | 3690 |
| ELET6 | 3690 |
| ITSA4 | 3690 |
| CMIG4 | 3690 |
| PETR4 | 3690 |
| VALE5 | 3690 |
| GGBR4 | 3421 |
| **TOTAL** | **29251** |

Source: Author's elaboration

The time to maturity of the option was calculated based on the difference in the days remaining between the date of the considered session and the due date stated in the contract. Based on the period of the selected sample and the assets identified as suitable by the study criteria, the options shown in Table 2 were selected. The pricing period consists of 18 business days (from October 22, 2014 until November 14, 2014). This period was chosen to enable the observation of the greatest number of price options from the market, given that the options have different issuance and expiration dates.

Table 2 – Set of options to be priced

| Role Code | Options Code | Issue | Expiration | Exercise Price | |
|---|---|---|---|---|---|
| USIM5 | USIML53 | 21/11/2014 | 14/12/2014 | R$ | 5.30 |
| ELET6 | ELETA11 | 17/11/2014 | 18/01/2015 | R$ | 7.50 |
| ITSA4 | ITSAL1 | 11/11/2014 | 14/12/2014 | R$ | 11.08 |
| CMIG4 | CMIGL75 | 13/11/2014 | 14/12/2014 | R$ | 14.14 |
| PETR4 | PETRL50 | 10/11/2014 | 14/12/2014 | R$ | 20.41 |
| VALE5 | VALEL47 | 05/11/2014 | 14/12/2014 | R$ | 17.85 |
| GGBR4 | GGBRL10 | 11/11/2014 | 14/12/2014 | R$ | 10.02 |

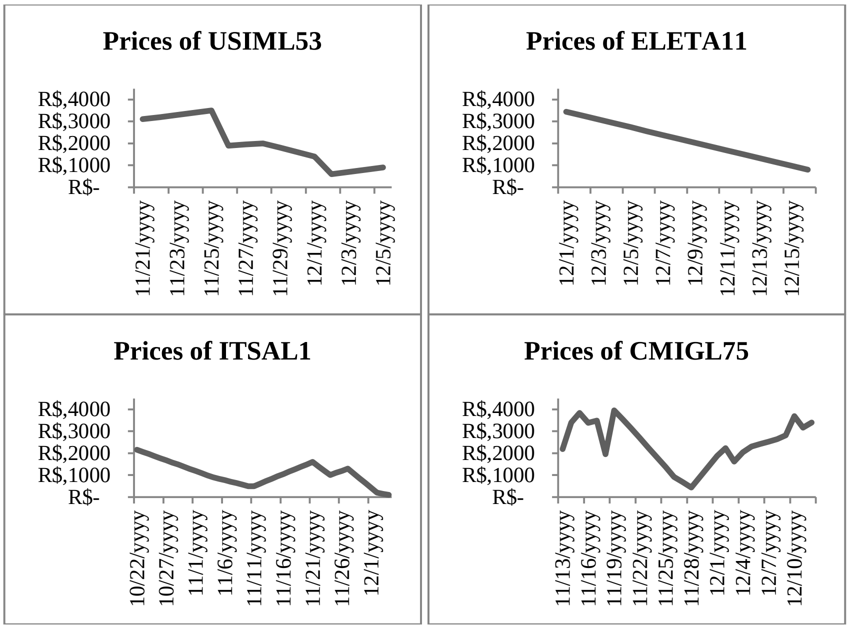
Source: Author's elaboration

The SELIC rate was assumed to be a good approximation of the risk-free Brazilian interest rate (SIMONASSI, 2006). The history of the series was removed from the database provided by the Central Bank of Brazil.

## 3.2. Empirical Model

The pricing models aim to estimate the premiums to minimize the differences in relation to the practiced and observed prices based on the market. Therefore, the model that will be identified as the one with the best performance will be that which presents the premium curve that best fits the prices used.

The model traditionally applied by the market (B-S) can be easily reproduced with the tools available in Excel. Many works have been developed to create practical manuals for applying the B-S model in this environment. This work will adopt the VBA code developed in the work by Rouah and Vainberg (2007). To apply the model by Heston, two software applications were used: Eviews (necessary for the implementation of the GARCH model and the short- and long-term analyses of asset volatility) and MATLAB (to estimate the model by Heston and identify the final results). The results will be compared with the behavior of the premiums of each option, the charts for which are illustrated below.



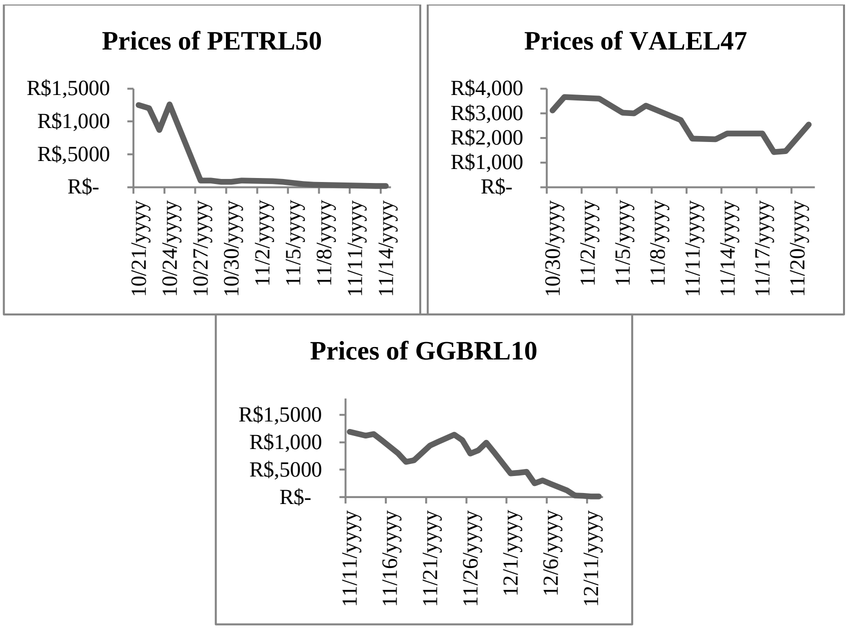



Figure 1 – Series of market prices

Source: Author's elaboration

## 3.3. Estimating the volatility of Heston: the GARCH model

In 2000, Heston and Nandi presented a form of options pricing combined with the use of a GARCH model. This new formula sought to review the stochastic volatility model in a time continuum, which had been submitted by Heston in 1993. The authors posit that in a daily database, the GARCH model allows for results that are numerically close to the stochastic time continuum model to be achieved, but with greater practicality. This advantage lies in the fact that the parameters of the GARCH model are easily estimated based on the observation of returns recorded in the underlying assets.

The charts below present the return on assets for which the options will be priced. The behavior of such returns is characterized by stationarity, which is a strong indication of autoregressive model applicability.

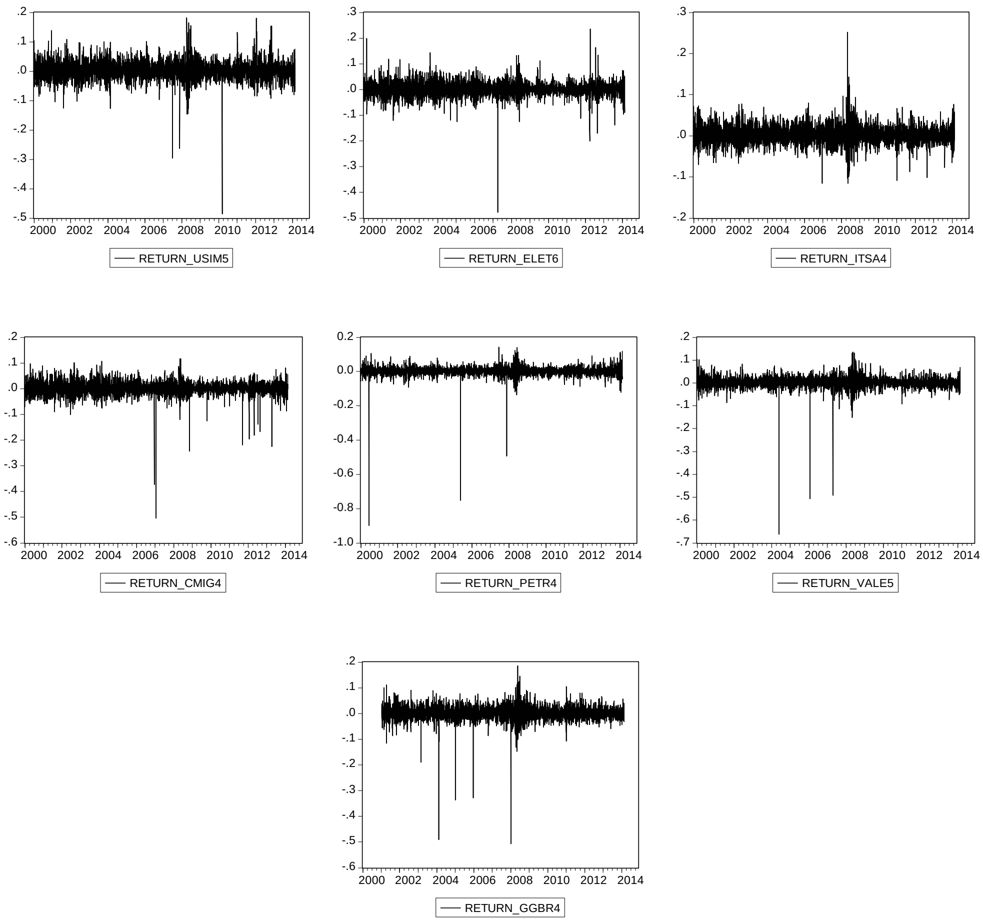



Figure 2 – Return on assets (2000 – 2014)

Source: Author's elaboration

Notably, the forecast horizon displays high relevance to the choice of pricing model, given that the relative importance among the recent and old observations is directly linked to the size of the target horizon. Other studies have found that the relative influence of old and recent observations will vary to the extent that the prediction horizon grows (EDERINGTON & GUAN, 2010 and MORAES, 2013). In the case of the GARCH model, the adoption of long-term forecasts may imply bias because models of this kind apply recursive substitutions that force the relative importance among observations (MORAES, 2013).

However, given that this study aims to design a short-term premium curve (less than one month), the adoption of the GARCH model will not involve this kind of problem, which was likewise observed by Heston and Nandi in their review work (HESTON and NANDI, 2000).

To identify the viability of the GARCH and General Exponential Autoregressive Conditional Heteroskedasticity (EGARCH) models, the ARCH test of heteroskedasticity was applied to all asset return series. The test of the presence of ARCH in the residuals was calculated based on the regression of the squared residuals at a constant with n lags, where n varies with each studied asset. To calculate the residuals, an ARMA (1,1) model was proposed. The test results enabled the evaluation of the presence of ARCH effects on the residuals from the returns on each stock studied, given that the test statistics provided by the *F Version* and *LM Statistic* are significant at 5%.

The Augmented Dickey Fuller (ADF) test was also applied to ensure that the series of return on the assets would be stationary. For all series of returns on the underlying assets, the null hypothesis of a unitary root was rejected, thus proving the stationarity of the data.

## 3.4. Testing the GARCH and EGARCH models

In some cases, the EGARCH model may appear as a valuable alternative for the prediction of volatility. The EGARCH model, also known as the exponential GARCH, may present more appropriate results according to the provision of the data to be analyzed.

In general, the EGARCH model is more appropriate than the GARCH model for cases that have persistent asymmetry because the model parameters are highly suitable for capturing this effect. For the actuality of this study, if the relationships between the volatility and the returns of the stocks are negative, the EGARCH model is expected to present the best results. To identify the inadequacy between the models and the exercise of forecasting the volatility of the selected stocks, the results of each model (GARCH and EGARCH) on the seven underlying assets were compared.

### 3.5. Comparative analysis of the results

The results were compared through an analysis of the prediction error measures. Among the possible measures are the mean squared error (MSE), the mean absolute error (MAE), and the mean absolute percentage error (MAPE). According to the methodology described above, the model that presents the lowest MSE, MAE, and MAPE values will be identified as the best forecasting model.

## 4. Results

After estimating the price curve through the B-S and Heston models (GARCH and EGARCH), the results were compared with the prices seen on the market. The results of the three tests chosen for selecting the best model (MSE, MAE, and MAPE) are shown in Table 3.

The model by Heston was superior to the traditional model by Black and Scholes, as reflected by the higher MSE, MAE, and MAPE values associated with the results obtained from the seminal model. In addition, the resulting similarity when comparing the GARCH and EGARCH models is noteworthy. These results reaffirm the common features of both models, although identifying a single optimal model remains difficult.

However, the difference observed in the options PETRL50 and GGBRL10 is notable. In the first case, the GARCH (1,1) underperformed relative to the EGARCH despite being superior to the B-S model. This result was expected because the GARCH parameter of the GARCH model (1,1) had no significant results in the preliminary tests. In the case of the option GGBRL10, the results of the MSE, MAE, and MAPE tests identified the GARCH (1,1) model as the best pricing alternative. This result was also expected because the asymmetry parameter of the EGARCH model is not significant at the level of 10% (see annex).

These results confirm the initial expectations of this work that stochastic models exhibit superiority in estimating the price curves of Brazilian options.

Table 3 – Results of the prediction error measures

| **Options Code** | **Test** | **B-S Model** | **Heston model (GARCH)** | **Heston model (EGARCH)** |
|---|---|---|---|---|
| USIML53 | MSE | 0,0064 | 0,0037 | 0,0032 |
| | MAE | 0,0691 | 0,0505 | 0,0438 |
| | MAPE | 47,5758 | 25,1075 | 24,0264 |
| ELETA11 | MSE | 0,0124 | 0,0040 | 0,0061 |
| | MAE | 0,0968 | 0,0474 | 0,0595 |
| | MAPE | 38,3871 | 14,3300 | 16,7254 |
| ITSAL1 | MSE | 0,0241 | 0,0037 | 0,0018 |
| | MAE | 0,0831 | 0,0408 | 0,0313 |
| | MAPE | 59,1202 | 25,6538 | 22,5167 |
| CMIGL75 | MSE | 0,0038 | 0,0028 | 0,0028 |
| | MAE | 0,0462 | 0,0390 | 0,0421 |
| | MAPE | 26,7852 | 16,6667 | 16,5000 |
| PETRL50 | MSE | 0,0316 | 0,1070 | 0,0316 |
| | MAE | 0,0985 | 0,1575 | 0,0861 |
| | MAPE | 94,9762 | 41,0625 | 20,3125 |
| VALEL47 | MSE | 0,2788 | 0,1391 | 0,0477 |
| | MAE | 0,4317 | 0,2946 | 0,1869 |
| | MAPE | 21,0080 | 11,5000 | 8,1017 |
| GGBRL10 | MSE | 0,0174 | 0,0097 | 0,0423 |
| | MAE | 0,0956 | 0,0682 | 0,1408 |
| | MAPE | 20,7813 | 10,8125 | 21,0938 |

Source: Author's elaboration

## 5. Conclusions

The results of this study confirm the limitations of the B-S model as an options pricing instrument. These limitations can mainly be attributed to the fact that the model adopts a simplistic and limited approach toward the behavior of the volatility of underlying assets. Nevertheless, the importance of the B-S model relative to the development of techniques currently used in the pricing of financial assets is undeniable.

Moreover, the Heston model appears as a more efficient alternative than the B-S model for considering stochastic volatility. This important differential ensures that the Heston model is an approach that is highly coherent relative to the reality experienced by the market. This model addresses problems such as the *volatility smile*, which underlies the main and most serious criticism against the assumptions made by Black and Scholes in 1973.

In addition, the stochastic model by Heston notably provides constant updating, although this feature has not been explored in this paper. By allowing the update of the projections with new market inputs, the Heston model can be expected to present highly efficient performance, thus expanding its differential as a pricing tool.

The results reaffirm the findings from other studies that have sought to compare the Heston model with the B-S model under the European and American market scenarios.

Further studies should adopt continuous updating procedures to test the gains observed relative to the performance by the Heston model. This study can also serve as an inspiration for new works that seek to investigate the applicability of the Heston model to American options traded in the Brazilian context. Other approaches may consider a stochastic interest rate and test the gains based on the obtained results.

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